Minimum-Snap Trajectory Generator with Error-State LQR Control

Andrew Torgesen[1]

[1]16.31 Final Project, Massachusetts Institute of Technology

Motivation

Experience has shown that quadrotor flight routines such as waypoint tracking can be satisfactorily executed with position and orientation-based successive loop closure. However, more acrobatic flight maneuvers which track continuous reference trajectories are better served with a control scheme that goes beyond position-based control. Better attitude control can be obtained by controlling off of both attitude and angular rate errors which have been derived from a high-fidelity, full-state trajectory generator and corresponding full error-state linear quadratic regulator.

This project aims to implement advanced quadrotor trajectory generation and control methods which leverage concepts such as **Lie theory**, **differential flatness**, and **quaternion attitude representation** in order to achieve **stable, acrobatic trajectory tracking** on the Parrot Mambo hardware platform.

Lie Theory Background

The attitude of rigid bodies cannot be represented as a vector. Thus, the dynamics of a quadrotor do not truly evolve over an affine space, but rather over a **manifold**, which is a nonlinear topology with continuous derivatives. Lie theory provides the mathematical tools for doing calculus on manifolds.

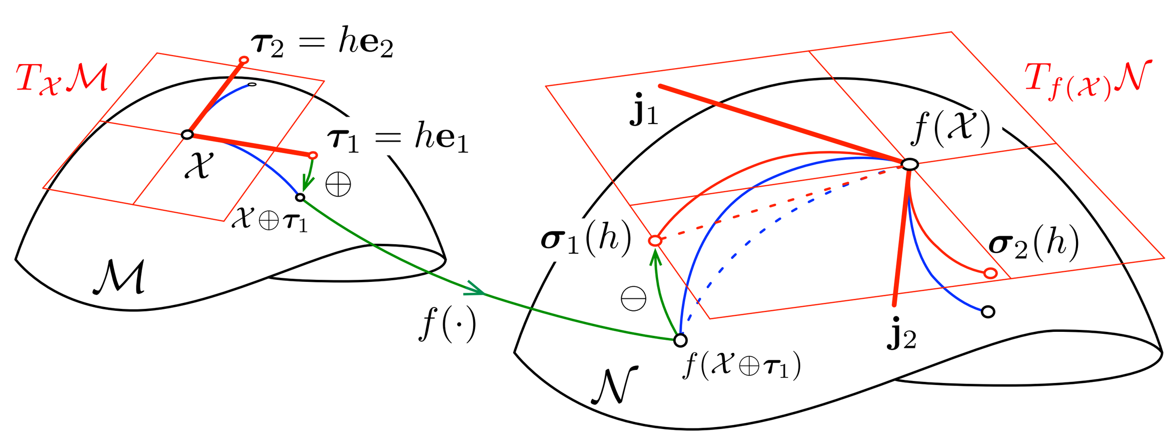

Figure 1. Illustration of the computations (from Lie theory) occurring on the manifold and its tangent spaces to compute the Jacobians of state dynamics for state representations that are not vector spaces. Figure taken from [3].

Lie theory facilitates calculus on manifolds by providing the following:

- A **bi-directional geodesic mapping** between a manifold, $\mathcal{M}$, and a space tangent to it, $T_{\mathcal{X}}\mathcal{M}$ (which is a vector space).
- A method for **adding** a vector increment to a manifold object with the $\oplus$ operator to evolve its state.
- A method for **subtracting** manifold objects with the $\ominus$ operator to compute a vector representing their difference.
- A **linear operator** (called the **adjoint**) for re-expressing a vector in a tangent space at manifold object $\mathcal{X}$ in terms of the tangent space at the identity manifold object (referred to as the **Lie Algebra**).

Project Description

The Simulink flight control system for the Parrot Mambo Minidrone is augmented with a full-state **T**rajectory **G**enerator and **E**rror-state **LQR** controller (**TG-ELQR**):

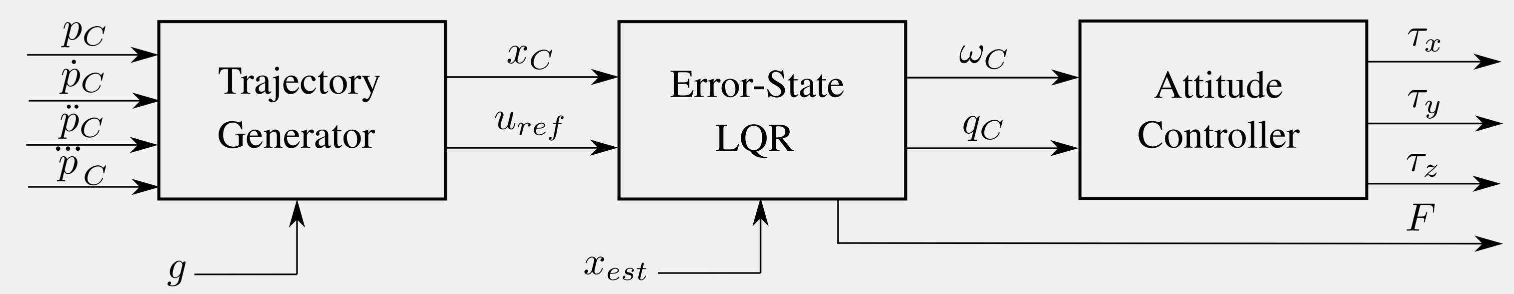

Figure 2. Architecture for augmented Parrot Mambo control system.

The trajectory generator, based primarily on the work laid out in [2], generates reference commands for every degree of freedom from the commanded values and derivatives of the four "differentially flat" states of position and yaw. The error-state LQR, based on [3] and [1], seeks to drive the state error to zero by performing control calculations directly on the manifold.

Simulation Results

Prior to hardware testing, a comparison was made between the position tracking performance of the Parrot Mambo's default controller and the TG-ELQR controller. The benchmark was a figure-eight flight pattern with smoothly varying position commands in all three dimensions.

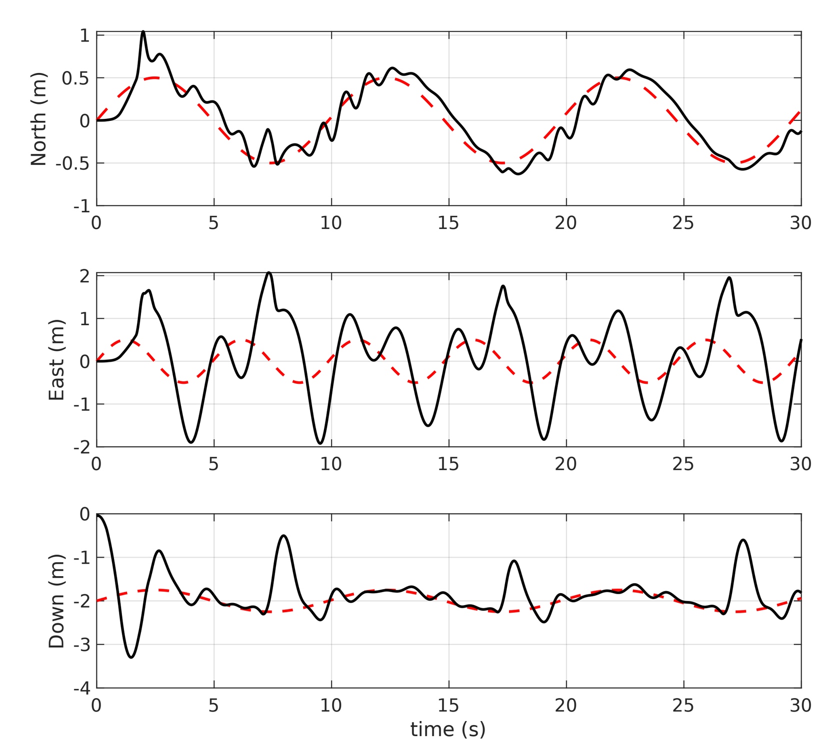

Figure 3. Default controller tracking.

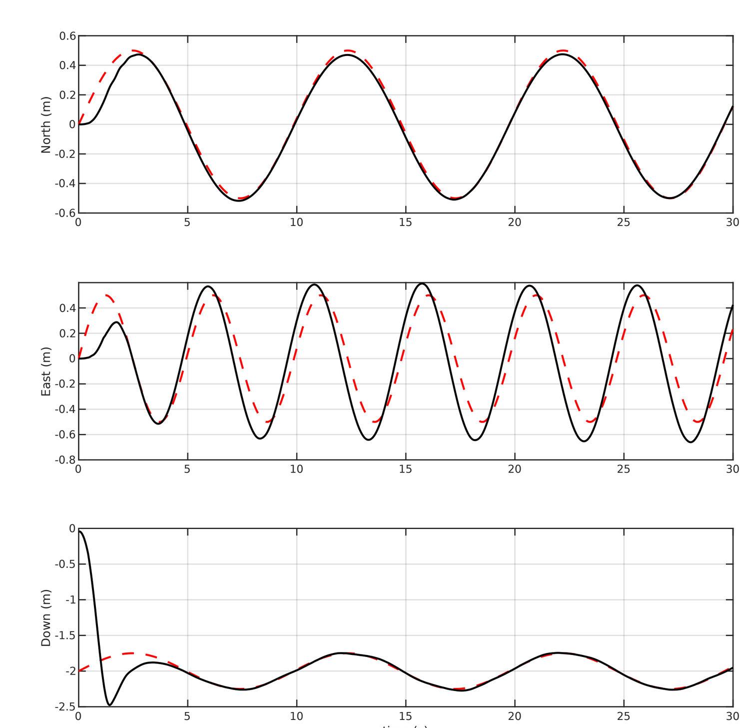

Figure 4. TG-ELQR controller tracking.

The significantly better tracking performance of the TG-ELQR controller points to the fact that the default controller for the Parrot Mambo is designed to achieve good performance in tracking step commands or waypoints, and not a continuously varying position command. The TG-ELQR controller, on the other hand, is able to convert smoothly varying position (and yaw) commands to full-state commands that can be efficiently tracked with full-state feedback for more agile flight performance.

Trajectory Generator

Quadrotor dynamics are **differentially flat**, meaning that given four key values $\begin{bmatrix} p_N & p_E & p_D & \psi \end{bmatrix}^T$ and their derivatives, all other state values can be derived from them algebraically.

The implemented trajectory generator assumes a constant yaw command of zero, and so it only takes the commanded position, velocity, acceleration, and jerk as its input, computing the remaining full-state trajectory command with the following calculations (which leverage the **exponential map** operator from Lie theory):

$$\theta = \cos^{-1}(e_3^T \frac{a}{||a||}) \qquad v_{b/I}^b = R_I^b \dot{p}_{b/I}^I \qquad p = h_\omega \cdot (R_I^b)^T e_2$$

$$q_I^b = \exp_q(\theta [e_3]_\times \frac{a}{||a||}) \qquad h_\omega = \frac{\dot{a} - ((R_I^b)^T e_3 \cdot \dot{a})}{||g - a||}(R_I^b)^T e_3 \qquad q = -h_\omega \cdot (R_I^b)^T e_1$$

$$r = 0$$

Error-State LQR

The error-state LQR controller is akin to normal LQR, with a few quirks:

- The state vector is defined as $\tilde{x} = x \ominus x_c$, such that $\tilde{x}$ (or the error-state) exists in the tangent space of the manifold that defines x.
- The A and B state space matrices come from the Jacobians of the **error-state dynamics**, rather than the nominal dynamics.
- Jacobians are calculated from the standard definition of the derivative, substituting the plus and minus operators with $\oplus$ and $\ominus$, respectively.

The error-state dynamics of a quadrotor are calculated to be

$$\dot{\tilde{p}}_{b/I}^I = (R_I^b)^T \tilde{v}_{b/I}^b - (R_I^b)^T [v_{b/I}^b]_\times \tilde{r}_I^b$$

$$\dot{\tilde{v}}_{b/I}^b = g[R_I^b e_3]_\times \tilde{r}_I^b - [\omega_{b/I}^b]_\times \tilde{v}_{b/I}^b + [v_{b/I}^b]_\times \tilde{\omega}_{b/I}^b$$

$$\dot{\tilde{r}}_I^b = \tilde{\omega}_{b/I}^b - [\omega_{b/I}^b]_\times \tilde{r}_I^b$$

References

[1] Michael Farrell, James Jackson, Jerel Nielsen, Craig Bidstrup, and Tim McLain. Error-state lqr control of a multirotor uav. pages 704–711, 06 2019.

[2] D. Mellinger and V. Kumar. Minimum snap trajectory generation and control for quadrotors. In *2011 IEEE International Conference on Robotics and Automation*, pages 2520–2525, May 2011.

[3] Joan Solà, Jérémie Deray, and Dinesh Atchuthan. A micro lie theory for state estimation in robotics. *CoRR*, abs/1812.01537, 2018.